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On Optimal British Columbia Log Export Policy: An Application of Trade theory

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Abstract

The log export policy suggestion by Dumont and Wright (2006) is critically assessed in an effort to determine if it is based on economic efficiency. The optimal log export policy for British Columbia is derived using two different models. The first model assumes that B.C. is a small open economy, and the second is a two country model that provides B.C. the opportunity to improve its terms of trade. In both cases it is shown that an optimal log export tax when a fixed lumber export tax exists can be characterized as a problem of second best. In that scenario the optimal log export policy is a positive export tax in both models. In the second model a positive export tax is also optimal when there is no lumber export tax, but it is smaller than when the lumber export tax is levied. In comparison to the log export tax recommended by Dumont and Wright (2006), the optimal tax for a small economy is always lower, while it is lower only in certain circumstances for an economy with market power. These results suggest that the Dumont and Wright policy is not efficient.

On Optimal British Columbia Log Export Policy: An Application of Trade Theory

1 Introduction

A consulting report to the provincial government of British Columbia in 2006 recommended a specific formulation for a tax on log exports (Dumont and Wright 2006). This report was the basis for changes in B.C. log export policy. The consultants report did not indicate whether or not the recommended tax was based on efficiency, distributional equity, or other grounds. The present study assesses the plausibility of the recommended tax being based on efficiecy arguments by deriving the efficient for models of the B.C. forest industry. In general the consultants' recommendation does not match the efficient taxes in the models.

The restriction of log exports has been one of the leading debates in British Columbia public policy throughout the province's history. Log exports have been restricted since the late 1800s and the evolution of policy has reflected the fundamental trade-off between efficiency and equity. Export restrictions reduce the value of the timber resource to those who own the right to harvest it, but improve the return on processing capital promoting investment and job creation in the processing sector (Shinn 1993). In pragmatic terms, restriction of log exports facilitates a transfer of wealth that bears distributional benefits. Those arguing for the removal of restrictions cite Samuelson's (1939) theorem that free-trade is a Pareto-optimal policy for a small price-taking economy if appropriate transfers can be made. If the hypotheses of Samuelson's theorem hold, then the transfer of wealth that occurs when log exports are restricted creates a net welfare loss. Whether the distributional change is worth its cost is a question without an objective answer, and no attempt to provide one is given in this study.

The literature is not without attempts to answer the aforementioned question. Margolick and Uhler (1986) determined that the net benefit to the B.C. economy of removing export restrictions would be $140 million, if the displaced 2640 workers are utilized elsewhere. Perez-Garcia *et al.* (1994) suggest that a log export ban in the United States Pacific Northwest (PNW) costs the owners of that region's timber resource $685 million per year. They estimate that this is mostly offset by gains made by lumber producers, but that the losses to consumers created by higher lumber prices imply net welfare losses. They further argue that a tax on log exports would have a similar effect, but that a tax might be more beneficial if revenues are channeled back to timber owners and trading partners do not retaliate. Similar results in a variety of contexts can be found in the literature (Sedjo and Wiseman 1981; Vincent 1989; Mani and Constantino 2004). These studies often argue for the removal of restrictions because of the welfare gains they imply. A different interpretation of those potential welfare gains is that they are the economic cost of of the distributional improvement created by restrictions. The costs of a tax on log exports would be direct transactions costs and possibly indirect costs from a shift in future behaviour due to altered incentives (Schwindt and Globerman 1996). Distorting taxes or fees on harvested timber lead to economically inefficient forest practices and reduce the value of timber harvesting rights (Paarsch 1993; Binkley and Zhang 1998). The difficult question to answer is whether or not the re-distribution justifies the costs.

Throughout most of the history of British Columbia the distributional benefits of restricting log exports have been deemed sufficient to justify the cost of lost economic efficiency. In 1888 an export tax on log exports was introduced for the first time, as was legislation making granted timber harvesting rights an appurtenance of manufacturing capacity. Three years later an outright ban on log exports was enacted. Since that time log exports have either been banned or allowed only under specific circumstances. These special circumstances have generally required that the logs be surplus to the requirements of provincial processing, or uneconomical to process within the province. In the present day, logs that are deemed to be surplus to provincial requirements are subject to an export tax that has been re-labeled a "fee in lieu of manufacture" (Shinn 1993).

The most recent development in British Columbia log export policy is the first of nine recommendations included in a consulting report to the B.C. provincial government (Dumont and Wright 2006). The report notes that British Columbia lumber exports to the United States are subject to an export tax, and that the lumber tax should be accompanied by an export tax on logs. Dumont and Wright wrote that such a tax would ensure "equivalent terms for the trade in lumber and logs," and "provide manufacturers with a level playing field relative to their U.S. competitors to process British Columbia logs." My interpretation is that, because lumber exports are penalized, the value of logs processed within the province is diminished. The decision of owners of timber harvesting rights is distorted toward exporting logs because the value of an exported log is not diminished. An export tax that reduces the value of an exported log by an amount equal to the reduction of the value of a provincially processed log would remove that distortion. Dumont and Wright suggest that the log export tax that accomplishes this is necessarily larger than the lumber export tax, a phenomenon they call the 'magnification effect.' The exact relationship between the lumber export tax, α, and the log export tax, $\overline{\tau}$, suggested by Dumont and Wright is:

$$\overline{\tau} = \frac{p_y}{p_x}\alpha, \tag{1}$$

where p_y and p_x are the domestic (British Columbia) prices of lumber and logs (per unit volume), respectively.[1] It is claimed that this mapping from the lumber export tax to the log export tax will ensure that the decision of those who own timber harvesting rights with regard to log exports will be identical to the case in which both taxes are zero.

By recommending the above log export tax, Dumont and Wright (2006) are asserting their belief that the implied transfer of wealth from the owners of timber harvesting rights to the processing sector will on net be beneficial to the province. It is not clear whether they believe there will be a net welfare gain, or whether it is the distributional change that justifies the tax. Put differently, it is not clear if the recommended tax would create a potential Pareto improvement or a distributional improvement. This question is central to the present study.

The large literature on optimal tariffs offers a good deal of suggestion regarding efficient export restrictions. As mentioned above, Samuelson's (1939) benchmark theorem states that if a country cannot alter its terms of trade (relative import and export prices) through tariffs, and appropriate re-distribution is possible, then free trade is a Pareto optimal policy. Sufficient conditions for appropriate re-distribution to take place are minimal, and it is safe to assume that assumption holds (Diewert, Turunen-Red and Woodland 1989). The other hypothesis, that an export tax will not alter British Columbia's terms of trade in logs, is debatable. Rather than attempting to resolve that debate here, both cases are examined in turn.

Suppose for the moment that B.C. is a price taker in international log markets. It is still not clear that no restriction on log exports is an optimal policy because free trade in all other goods may not be possible. Samuelson's theorem tells us that free

[1]Dumont and Wright (2006) interpret p_y as the 'lumber mill net,' or the price received by manufacturers net of transportation costs, tariffs, etc. They interpret p_x as the cost of delivered logs in province. They actually suggest $\overline{\tau} = \alpha/(p_x/p_y)$, but the equivalence with equation (1) is obvious.

trade in all other goods is Pareto efficient, but says nothing about efficiency if there is a fixed tariff on one of the goods. A host of literature exists on the subject of optimal policy in the case of a fixed tariff on one good, some of which is focused on optimal policy regarding intermediate goods when the final good is subject to tariff. In the forest industry logs are an intermediate good in the production of lumber, and thus a review of that literature is instructive.

Contributions to the literature regarding optimal tariffs on intermediate goods usually suppose a particular mixture of imports and exports of intermediate and final goods. Ruffin (1969) analyzes the case of a small price-taking economy that domestically produces and imports two final goods, x_1 and x_2, and imports an intermediate good, x_3. The intermediate good is used in fixed proportions in the production of x_2: $x_2 = ax_3$. Ruffin shows that if an arbitrary tariff, α, is imposed on imports of x_2, the optimal tariff, τ, on imports of x_3 is

$$\tau = \frac{ap_{x_2}}{p_{x_3}}\alpha. \tag{2}$$

The economy improves its welfare when the tariff on x_3 is imposed because, in its absence, the tariff on x_2 causes domestic producers to shift resources from x_1 to x_2. The tax on the intermediate good as given in equation (2) shifts resources back to production of x_1, so that production the three goods is identical to that under a free-trade equilibrium. The efficient policy, given that α is fixed, is to tax imports of the intermediate good. Because the tariffs offset each other, resulting in free-trade resource allocation, the solution is a zero effective rate of protection. This result is confirmed by Casas (1973).[2]

If a country is able to improve its terms of trade by imposing a tariff - i.e. it

[2]Casas (1973) points out that this does not hold if the fixed tariff is on the intermediate good and the flexible tariff is on a final good.

increases the price of its exports relative to its imports - it can improve its welfare by doing so (Krueger and Sonnenschen 1967). This notion is very similar to the notion that monopolists can increase their profit by restricting output and raising their price. To generate the necessary terms of trade effects previous authors have used two country general equilibrium models, where one country is referred to as “home” and the other as “foreign.” Das (1983) shows that in such a model the presence of a fixed tariff on an imported final good, the optimal tariff on imports of an intermediate good that is used in domestic production of the restricted final good can be negative, positive or zero under certain conditions. A similar result is derived by Suzuki (1978), in the context of an export tax rather than import tariff.

Another of Dumont and Wright’s (2006) recommendations may have a bearing on the subject of this paper, namely that free trade in logs be provided as a concession for free trade in lumber. If free trade in lumber could be achieved in this manner, the optimal log export policy could be much different from that derived in this paper. It has been shown that, in a static two country two good game theoretic trade model, the unique non-cooperative equilibrium is for both countries to levy tariffs, while the cooperative equilibrium is free trade (Dixit 1987, McMillan 1986). In more elaborate sequential models, in which the leading country is uncertain about the following country’s reaction, Thursby and Jensen (1983, 1990) have shown that an increased likelihood of retaliation, or uncertainty in general, results in lower equilibrium tariffs. In that sense there may be some export restriction commitment by B.C. that could make free trade with the United States mutually beneficial. A good discussion of B.C.’s willingness to have lumber export restrictions removed is provided by Niquidet (2008).

Recent literature has been attentive to the environmental benefits of log export restrictions, arguing that reduction of log exports implies a reduced harvest that enhances the non-market values of forests (Barbier, Burgess, Ayward and Bishop 1992;

Burgess 1993; Goodland and Daly 1996). In the context of B.C. this argument is weak given that the harvest of timber in Canada and the United States is determined by the maximum sustainable harvest from industrial forests, also called the Annual Allowable Cut (AAC) (Uhler 1991). It can occur that utilization of timber and appropriate forest practices are adversely impacted by lower log prices. If this is the case the environment could be negatively affected by log export restrictions. However, for the purposes of this study, it will be assumed that harvest and forest practices in Canada and the United States are fixed by the AAC and other regulations, rendering the environment irrelevant in log export policy analysis.

In this paper, the optimal log export policy for British Columbia is derived using two different models. Distributional benefits are ignored in the definition of optimal. The first model assumes that B.C. is a small economy that cannot change its terms of trade, and the second is a two country model that provides B.C. the opportunity to improve its terms of trade. In both cases it is shown that an optimal log export tax when a lumber export tax exists can be characterized as a problem of second best. In that scenario the optimal log export policy is a positive export tax in both models. In the second model a positive export tax is also optimal when there is no lumber export tax, but it is smaller than when the lumber export tax is levied. In comparison to the log export tax recommended by Dumont and Wright (2006), the optimal tax for a small economy is always lower, while it is lower only in certain circumstances for an economy with market power. These results suggest that the Dumont and Wright policy is not an optimal one.

This paper proceeds as follows. Section two develops a small economy model, derives optimal log export policy and compares it to the Dumont and Wright policy; section three develops a two country model and proceeds as in section two; section four provides an empirical example; and section five discusses and concludes.

2 British Columbia as a Small Economy

To consider provincial welfare meaningfully in the context of log exports, a model with several key characteristics must be formulated. The model must have the utility of residents as its objective function, and their consumption possibilities must depend on income derived from the forest industry. It is also necessary to deal with Arrow's (1950) social welfare problem. To satisfy these needs a model very similar to that of Ruffin (1969) is formulated, in which a single representative agent owns the means of production and the timber resource. By doing so the analysis focuses on economic efficiency and leaves aside the problem of distributional benefits.

The province is modeled by a single representative agent who consumes an imported numeraire good, m, and lumber, y_d, and has increasing quasiconcave utility $U(m, y_d)$. The agent owns the province's timber harvest, $\overline{x}$, and its lumber producing capital. The agent's income is derived from selling logs and lumber abroad. Exported logs and lumber are then given by $\tilde{x} = \overline{x} - x$ and $\tilde{y} = y - y_d$, respectively. By assumption $\tilde{x} > 0$ and $\tilde{y} > 0$.

Let $F : (x, z) \rightarrow y$ be the production function of lumber, y, from logs, x, and other production requirements, z. These other requirements can be thought of as a bundle containing the labour, electricity, etc. that is needed to produce a quantity of lumber.[3] Since both x and z are required to produce lumber they are perfect compliments in the production process and $F(x, z) = \min(f(x), g(z))$. In an equilibrium it will be the case that $f(x) = g(z) = y$, otherwise there will be waste.

The mapping of logs to lumber, f, is a linear funtion. The derivative, $f'(x) \in [0, 1]$, is often referred to as the lumber recovery factor.[4] It is determined by the capital

[3]One could also replace z with a vector of requirements for greater generality, but this would have no bearing on results.

[4]If the price of of by-products is held fixed their inclusion as a valuable output has no bearing on the results and for that reason they are ignored in this analysis.

stock, which is assumed fixed in this analysis, thus $f''(x) = 0$. The mapping of other production factors to lumber, g, is concave and increasing, such that on net there are decreasing returns to scale. The cost of a unit of the input bundle, z, is w. Since $y = g(z)$ in equilibrium, the cost of z can be written as $c(y) = wg^{-1}(y)$. Since g is concave and increasing, g^{-1} will be convex and increasing, which implies that $c(y)$ is convex and increasing as well. For the remainder of this essay I simply work with $c(y)$.

Logs and lumber are traded in international markets at prices $\hat{p}_x$ and $\hat{p}_y$, respectively. In the absence of trade barriers these equate to domestic prices. If trade barriers exist in the form of ad-valorem export taxes, τ on logs and α on lumber, the relationships between domestic and international prices will be $p_x = (1-\tau)\hat{p}_x$ and $p_y = (1-\alpha)\hat{p}_y$. Any revenue from export taxes returned to the representative agent is in the form of an exogenous lump sum payment T.

2.1 The Private Optimum

The representative agent's problem is maximize $U(m, y_d)$ with respect to $\{m, y_d, x\}$ given prices, export taxes and transfers, subject to

$$m + p_y y_d \leq f(x)p_y + (\overline{x} - x)p_x - c(y) + T. \tag{3}$$

The necessary conditions for a private optimum are:

$$\frac{U_{y_d}}{U_m} = p_y \tag{4}$$

$$p_x = f'(x)\left[p_y - c'(y)\right], \tag{5}$$

and equality in equation (3).[5] These conditions implicitly define the private allocation: (m^*, y_d^*, x^*).

Equations (4) and (5) are written in terms of domestic prices, which are identified by the corresponding domestic-international price relationships given above. The implication is that the representative agent's private optimum is flexible, and their behaviour adjusts to export policy. In the absence of an export tax on lumber ($\alpha = 0$), the agent's optimum requires that their marginal rate of substitution be set equal to $\hat{p}_y$, but if a lumber export tax is levied marginal utility must shrink to $(1 - \alpha)\hat{p}_y$. A similar argument can be made regarding the second necessary condition, equation (5), but there is an important difference. If α were to increase, the representative agent would reduce their input of logs so that $c'(y)$ decreases and the right hand side of (5) equates with p_x. This assumes that τ is fixed; however, if it is flexible and responsive to changes in α, log input may not need to be adjusted for equation (5) to hold. This possibility is in fact the basis for the optimal log export policy derived below, but another result, pertaining to the social optimum, is needed to complete the derivation.

2.2 *Efficient Policy and the Problem of Second Best*

To determine what can be considered optimal export policy, the notion of a benevolent social planner is introduced. The problem faced by the planner is distinguished from that of the private agent by a key difference: the planner is constrained by what is economically feasible, whereas the representative agent is constrained by their budget. Unlike an individual's budget set, the economically feasible set is not altered by export policy because any revenue from taxes can be transferred back to the consumer. The planner seeks to maximize the welfare of the agent subject to what is economically

[5]The quasi-concave objective and convex constraint imply that the second-order condition for a maximum is met. For completeness Appendix 1 proves that sufficient conditions for this and all other optimization problems in this paper hold.

feasible, and their solution is the economically efficient allocation.

Specifically, the social planner's problem is then to maximize $U(m, y_d)$ with respect to $\{m, y_d, x\}$ subject to

$$m + \hat{p}_y y_d \leq f(x)\hat{p}_y + \tilde{x}\hat{p}_x - c(y). \tag{6}$$

Equation (6) differs from equation (3) in that the prices are international rather than domestic and there are no transfers. This is because the planner is constrained by the economy's feasible set, which is unaffected by taxes and transfers, unlike the representative agent's budget set. The first-order necessary conditions for this problem are

$$\frac{U_{y_d}}{U_m} = \hat{p}_y \tag{7}$$

$$\hat{p}_x = f'(x)\left[\hat{p}_y - c'(y)\right], \tag{8}$$

and equality in equation (6). Equations (7) and (8) are then necessary conditions for an efficient allocation, denoted $(m^\diamond, y_d^\diamond, x^\diamond)$, also referred to as the first best allocation.

The first best serves as a normative benchmark. Its use arises in comparison against the private outcome, derived from positive theory. If the private outcome matches the first best allocation, then the private outcome is efficient. If policy causes the private outcome to be shifted from any allocation to the first best allocation, then the representative agent is no worse off.

In the present model, the necessary conditions for a private outcome coincide with with those of the first best outcome if domestic and international prices equate. The fact that this occurs when export taxes are zero is the coveted theorem that free-trade is economically efficient. This result is so well known that it is not worth belaboring with formal treatment here. Rather, an important related point is presented.

Proposition 1. *Suppose that $\alpha > 0$, then $U(m^*, y_d^*) < U(m^\diamond, y_d^\diamond)$.*

Proof. A necessary condition for (m^*, y_d^*, x^*) is $U_{y_d}/U_m = (1-\alpha)\hat{p}_y \neq \hat{p}_y$ if $\alpha > 0$. Then $U(m^*, y_d^*) \leq U(m^\diamond, y_d^\diamond)$. To prove strict inequality a Lagrangian is formed to solve the representative agent's problem, and it is evaluated at (m^*, y_d^*, x^*). Differentiating with respect to α and simplifying using the necessary conditions yields:

$$\partial_\alpha U(m^*, y_d^*) = \lambda \left[\partial_\alpha x^* p_x - \hat{p}_y \tilde{y}\right].$$

By applying the implicit function theorem to equation (5) it can be shown that the partial derivative $\partial_\alpha x^* = -f'(x)c''(y)/\hat{p}_y < 0$, since $c(y)$ is convex. Then $\partial_\alpha U(m^*, y_d^*) < 0$. Since $(m^*, y_d^*, x^*) = (m^\diamond, y_d^\diamond, x^\diamond)$ only if $\alpha = 0$, $U(m^*, y_d^*) < U(m^\diamond, y_d^\diamond)$ when $\alpha > 0$ since U is strictly decreasing in α. $\square$

The point of Proposition 1 is that the first best outcome is unattainable if there is a restriction on lumber exports. The theorems that tell us that free-trade results in the first best allocation are useless as that allocation cannot be achieved. The question that must now be addressed is: given that α is positive, what is the solution to the social planner's problem? This is referred to as the problem of second best. If the efficient allocation cannot be achieved, what is the best that *can* be achieved.

In determining the second best allocation Ruffin's (1969) "Fundamental Lemma" will be very useful. A version of that Lemma, modified to fit the current model, is presented.

Lemma 1 (Ruffin's)**.** *If $U_{y_d}/U_m = k\hat{p}_y$, where $k \neq 1$ is a constant, then a necessary condition for a second best optimal is:*

$$\hat{p}_x = f'(x)\left(\hat{p}_y - c'(y)\right). \tag{8}$$

Proof. The social planner's problem is to maximize $U(m, y_d)$ subject to equation (6) and $U_{y_d}/U_m = k\hat{p}_y$, or equivalently:

$$\max_{\{m,y_d,x\}} L^p = U(m, y_d) + \lambda_1 \left[f(x)\hat{p}_y + \tilde{x}\hat{p}_x - c(y) - m - \hat{p}_y y_d\right] + \lambda_2 \left[k\hat{p}_y - (U_{y_d}/U_m)\right].$$

The first-order necessary conditions for a maximum are equality in the constraints and:

$$\nabla L^p = \begin{pmatrix} U_m - \lambda_1 - \lambda_2 \left(U_{y_d m}U_m - U_{mm}U_{y_d}\right)/U_m^2 \\ U_{y_d} - \hat{p}_y\lambda_1 - \lambda_2 \left(U_{y_d y_d}U_m - U_{m y_d}U_{y_d}\right)/U_m^2 \\ \lambda_1 \left(f'(x)\hat{p}_y - \hat{p}_x - f'(x)c'(y)\right) \end{pmatrix} = \mathbf{0}.$$

The third condition implies equation (8), since $\lambda_1 \neq 0$ unless there is waste. $\square$

2.3 Optimal Log Export Policy

Given Ruffin's Lemma, the optimal log export can be determined. For completeness and clarity a formal definition is given, followed by the result. Denote by (m^s, y_d^s, x^s) the second best optimal allocation.

Definition An exogenously chosen optimal log export tax, τ^*, is such that, given $\alpha > 0$, $(m^*, y_d^*, x^*) = (m^s, y_d^s, x^s)$, and, given $\alpha = 0$, $(m^*, y_d^*, x^*) = (m^\diamond, y_d^\diamond, x^\diamond)$ so that $U(m^*(\tau^*), y_d^*(\tau^*)) \geq U(m^*(\tau), y_d^*(\tau))\ \forall\tau$.

Implicit in this definition is the assumption that the private optimum can coincide with the second best optimal. This can generally be achieved in the model, although a closed form solution would require specific functional forms.

Theorem 1. *For a small economy as described and an arbitrary lumber export tax α*

the optimal log export policy is

$$\tau^* = \frac{f'(x)\hat{p}_y}{\hat{p}_x}\alpha. \tag{9}$$

Proof. For the case $\alpha > 0$, by the definition of an optimal log export policy, equations (5) and (8) must be satisfied because they are necessary for a private optimum and a second best optimum, respectively. Equation (5) can be rearranged into:

$$\tau = \frac{1}{\hat{p}_x}[\hat{p}_x - f'(x)(\hat{p}_y - c'(y))] + \frac{f'(x)\hat{p}_y}{\hat{p}_x}\alpha. \tag{5a}$$

When equation (8) is satisfied, the first term of the RHS of (5a) is zero, and $\tau = \tau^*$ if equation (5) is satisfied. Now, suppose $\tau \neq \tau^*$, then equation (5) cannot be satisfied when equation (8) is.

For the case $\alpha = 0$, equations (4) and (7) are satisfied always. If $\tau = \tau^*$ (equals zero if $\alpha = 0$) then equations (5) and (8) are both satisfied. If $\tau \neq \tau^*$ then equation (4) cannot be satisfied if equation (8) is. □

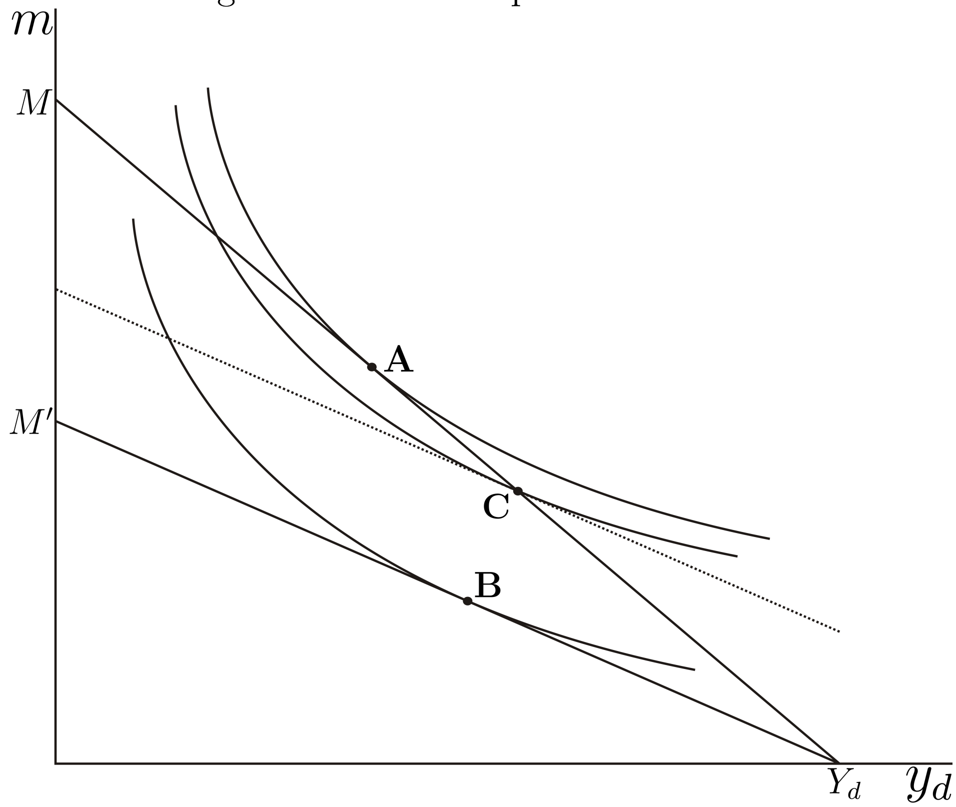
This result says that the optimal log export policy will ensure equation (8) is satisfied regardless of whether or not equation (7) is satisfied. It is not immediately obvious why this is the case, but a more intuitive graphical analysis is instructive.

Figure 1 illustrates graphically the optimization problems discussed above. Consider first the consumer's optimization problem, which was to maximize utility subject to equation (3), re-written as equation (3a).

$$m = f(x)p_y + (\overline{x} - x)p_x - c(y) + T - p_y y_d. \tag{3a}$$

Notice that the slope of equation (3a) is p_y, but that the intercept depends on x. Maximizing the intercept with respect to x ensures that the budget set is as large as

Figure 1: Various Optimal Solutions



possible. Mathematically, this is ensured by equation (5).

For the case $\alpha = \tau = 0$, the budget line is represented in figure 1 by the line MY_d. The private optimum is located at point **A**, where $U_{y_d}/U_m = \hat{p}_y$. For the case $\alpha > 0$, the budget line is represented by $M'Y_d$. Examination of equation (3a) indicates that when α rises the m intercept unambiguously falls, the slope becomes flatter and the y_d intercept is no larger.[6] The private optimal in this case is at point **B**, where $U_{y_d}/U_m = (1-\alpha)\hat{p}_y$.

Now consider the planner's problem. When $\alpha = \tau = 0$ it is identical to the consumer's problem, and the efficient allocation is at point **A**. However, when $\alpha > 0$, the planner's problem differs from the agent's. The allocation chosen by the planner must satisfy two constraints: it must be on the consumption possibilities frontier (MY_d),

[6] Y_d will fall if some or all of the cost of manufacturing consumed lumber is paid by exporting lumber. If costs are covered by exporting logs, then Y_d is unaffected.

and the consumer's indifference curve must be *parallel* to their budget constraint. The planner's choice will be the second best optimal, and will be located at point **C** in figure 1. This point is on MY_d and the consumer's indifference curve is tangent to the dashed line, which is parallel to $M'Y_d$. The optimal log export policy is in effect the τ that maximizes the intercept of $M'Y_d$, which equates it to the dashed line.

2.4 The Optimal and Dumont and Wright's Policy

By computing the optimal log export policy in Theorem 1, a normative benchmark against which to compare alternative policies has been created. Recall that Dumont and Wright (2006) recommended that the B.C. log export tax should be $\overline{\tau}$ as given in equation 1. At first glance there is a similarity in that both $\overline{\tau}$ and τ^* equate to zero if there is no export tax on lumber, meaning both reflect the wisdom that free trade is optimal if it can be achieved. If $\alpha > 0$, however, comparisons become difficult because $\overline{\tau}$ is a function of domestic prices and τ^* is a function of international prices. Dumont and Wright's recommendation is a quadratic function when expressed in terms of the international prices. This could be solved, however, given one more assumption a meaningful comparison of $\overline{\tau}$ and τ^* can be made by estabishing bounds on $\overline{\tau}$.

Proposition 2. *Suppose that $f'(x) < (1-\alpha)$ and $\alpha > 0$, then $\alpha < \tau^* < \overline{\tau}$.*

Proof. Substitution of the domestic-international price relationships into equation (1) yields $\overline{\tau}(1-\overline{\tau}) = (\hat{p}_y/\hat{p}_x)\alpha(1-\alpha)$, which implies the following inequality:

$$(1-\alpha)\frac{\hat{p}_y}{\hat{p}_x}\alpha < \overline{\tau} < \frac{1}{1-\overline{\tau}}\frac{\hat{p}_y}{\hat{p}_x}\alpha.$$

The assumption $f'(x) < (1-\alpha)$ implies $\tau^* = f'(x)(\hat{p}_y/\hat{p}_x)\alpha < \overline{\tau}$. To estimate from below note that $f'(x)\hat{p}_y - f'(x)c'(y) = \hat{p}_x$, which implies $f'(x)\hat{p}_y > \hat{p}_x$ since the cost function is assumed to be strictly increasing. □

Remark Empirically we observe that the lumber recovery factor ($f'(x)$) is less than 1/2, while the export tax on lumber exports never exceeds 1/2. This suggests that the additional hypothesis is true.

Corollary 1. *If $f'(x) < (1-\alpha)$ and $\alpha > 0$, then $\overline{\tau} \neq \tau^*$.*

Proof of Corollary 1 is redundant, but the result is important. Dumont and Wright's (2006) recommended tax is not the optimal log export policy whenever there is a positive export tax on lumber. Proposition 2 says that $\overline{\tau}$ would result in over taxation of log exports, resulting in sub-optimal domestic lumber manufacturing. Also from Proposition 2, the "magnification effect" discussed by Dumont and Wright does exist - the optimal log export tax is always larger than the imposed lumber export tax.

3 British Columbia with market power in logs

To create the potential for an economy to improve its terms of trade, the literature has generally used two economy general equilibrium models. The approach here is similar, but a simplifying assumption is made: all markets but logs are internationally competitive. That is, in equilibrium the total supply of logs by the two economies must equate to their total demand for logs, but for other goods excess supply or demand will be cleared by international markets at fixed prices.

The major foreign buyer of British Columbia logs is the United States Pacific Northwest (PNW). The abstract economies of the model, home and foreign, are intended to mimic these regions, but they are certainly not exact replications of them. The notation for a variable in the foreign country is a 'hat,' for example $\hat{y}$ is foreign lumber production. Like the small economy model, $\hat{p}_y$ is the international price of lumber, but $\hat{p}_x$ is re-defined slightly as the foreign price of logs. Exports from home to foreign are denoted by a 'tilde,' such that $\tilde{x}$ is log exports from home to foreign. The home-foreign price relationships are as given in §2: $(1-\tau)p_x = \hat{p}_x$ and $(1-\alpha)p_y = \hat{p}_y$. It assumed that the home and foreign economies have identical lumber manufacturing capital and cost functions, implying that $f'(x) = f'(\hat{x})$. The home economy has a larger timber harvest than the foreign economy, $\overline{x} > \hat{\overline{x}}$.

In addition to assuming the cost function is the same in both regions, it is given some additional restrictions. The second derivative of $c(y)$ is assumed to be constant, which implies that higher-order derivatives are zero. Furthermore, the marginal cost function $c'(y)$ is assumed linear so that $c'(y) = yc''(y)$. Though restrictive, these assumptions make the model far more tractable, and the results are more easily interpreted.

3.1 Competitive General Equilibrium

The timber harvest and processing capital of each economy is owned by a representative agent for that economy. The home agent has preferences $U(m, y_d)$ and the foreign agent has preferences $\hat{U}(\hat{m}, \hat{y}_d)$. Agents maximize utility subject to equations (10) and (11), the home and foreign budget sets, respectively.

$$
\begin{aligned}
m + p_y y_d &\leq f(x)p_y + \tilde{x}p_x - c(y) + T && (10)\\
\hat{m} + \hat{p}_y \hat{y}_d &\leq f(\hat{x})\hat{p}_y - \tilde{x}\hat{p}_x - c(\hat{y}) && (11)
\end{aligned}
$$

No transfers are included in the foreign constraint as no trade restrictions are considered there. Because the economies are assumed to be competitive, agents take their respective domestic prices of logs as given.

The first-order conditions for utility maximization are given by equations (12) and (13) for the home agent, and equations (14) and (15) for the foreign agent.

$$
\begin{aligned}
\frac{U_{y_d}}{U_m} &= p_y && (12)\\
p_x &= f'(x)\left(p_y - c'(y)\right) && (13)\\
\frac{\hat{U}_{y_d}}{\hat{U}_m} &= \hat{p}_y && (14)\\
\hat{p}_x &= f'(x)\left(\hat{p}_y - c'(\hat{y})\right) && (15)
\end{aligned}
$$

The equations implicitly define the agents' optimal feasible bundles (m^*, y_d^*, x^*) and $(\hat{m}^*, \hat{y}_d^*, \hat{x}^*)$. General equilibrium in the model is defined, in part, using these equations.

Definition A general equilibrium in the two country model, given an international lumber price $\hat{p}_y$ and export taxes τ and α on $\tilde{y}$ and $\tilde{x}$, respectively, is a log price $\hat{p}_x^e$ and a quantity of log exports $\tilde{x}^e$ such that $x + \hat{x} = \overline{x} + \hat{\overline{x}}$ and agents in both economies maximize utility.

Proposition 3. *If $\tau = \alpha = 0$, $\hat{p}_x^e > 0$ and $\tilde{x}^e > 0$, then if the two country model is in general equilibrium $\tilde{x}^e = \frac{1}{2}\left(\overline{x} - \hat{\overline{x}}\right) > 0$.*

Proof. If $\alpha = 0$ equations (12) and (14) are satisfied always. When $\tau = 0$ and $\tilde{x}^e > 0$, the right hand sides of (13) and (15) must equate implying $c'(\hat{y}) = c'(y)$. Since $c'(y)$ is linear, this can be written as $f(\hat{x})c''(\hat{y}) = f(x)c''(y)$, which can be further simplified to $(\hat{\overline{x}} + \tilde{x}^e) = (\overline{x} - \tilde{x}^e)$ using the linearity of the production function. This implies $\tilde{x}^e = \frac{1}{2}\left(\overline{x} - \hat{\overline{x}}\right) > 0$ if (13) and (15) are satisfied, and these are necessary for general equilibrium. This $\tilde{x}^e$ satisfies $x + \hat{x} = \overline{x} + \hat{\overline{x}}$. Since $\overline{x} > \hat{\overline{x}}$ it follows that $\frac{1}{2}\left(\overline{x} - \hat{\overline{x}}\right) > 0$. $\square$

Remark The assumption $\overline{x} > \hat{\overline{x}}$ is required to ensure $\tilde{x}^e > 0$.

Equation (15) can be interpreted as the foreign country's willingness to pay for logs. Examination of that equation indicates that the foreign willingness to pay for logs - and log imports - is downward sloping. No account of this is taken in the definition of general equilibrium because the economies are competitive. However, an optimal home economy log export policy will take account of the downward sloping demand for log exports. In doing so it will be useful to know how the general equilibrium price and quantity of log exports will change with changes in export taxes. An explicit solution cannot be attained in this model, but the derivatives can be attained implicitly.[7] To do this, the function $\mathbf{G} : \mathbb{R}^j \to \mathbb{R}^2$, where $j > 2$ are the general equilibrium par

$$\mathbf{G} = \begin{pmatrix} (1-\tau)\hat{p}_x^e - f'(x)\left[(1-\alpha)\hat{p}_y - c'(f(\overline{x} - \tilde{x}^e))\right] \\ \hat{p}_x^e - f'(x)\left[\hat{p}_y - c'(f(\hat{\overline{x}} + \tilde{x}^e))\right] \end{pmatrix} = \mathbf{0}$$

[7] A full and careful explanation of the method used here can be found in Folland (2002, p.118).

The Fréchet derivative of $\mathbf{G}$ with respect to $\mathbf{a} = (\tilde{x}^e, \hat{p}^e_x)$ and $\mathbf{b} = (\tau, \alpha)$ is:[8]

$$D_{(\mathbf{a},\mathbf{b})}\mathbf{G} = \begin{pmatrix} (1-\tau) & -f'(x)c''(y) & -\hat{p}^e_x & -f'(x)\hat{p}_y \\ 1 & f'(x)c''(y) & 0 & 0 \end{pmatrix}.$$

The determinant of the sub matrix $D_{\mathbf{a}}\mathbf{G}$ is non-zero if $\tau \neq 2$. Thus, by the Implicit Function Theorem, there are functions,

$$\left.\begin{array}{rcl} \tilde{x}^e & = & h(\mathbf{b}) \\ \hat{p}^e_x & = & h(\mathbf{b}) \end{array}\right\} \mathbf{a}^T = \mathbf{H}(\mathbf{b})$$

whenever $\tau \neq 2$. The derivatives of the general equilibrium solution with respect to export taxes are

$$D_{\mathbf{b}}\mathbf{H} = -[D_{\mathbf{a}}\mathbf{G}]^{-1}D_{\mathbf{b}}\mathbf{G} = \begin{pmatrix} \frac{\hat{p}^e_x}{(2-\tau)} & \frac{-f'(x)\hat{p}_y}{(2-\tau)} \\ \frac{-\hat{p}^e_x}{(2-\tau)f'(x)^2c''(y)} & \frac{f'(x)\hat{p}_y}{(2-\tau)f'(x)^2c''(y)} \end{pmatrix}. \tag{16}$$

Not all that surprisingly, $\hat{p}^e_x$ is increasing in τ and decreasing in α, and $\tilde{x}^e$ behaves oppositely.

3.2 Optimality and the Problem of Second Best

Care must be taken in choosing a normative benchmark in this model. In a strict sense, economic efficiency implies that the total welfare of the home and foreign agents together is maximized. In the present model efficiency is not achieved if the home country exercises its international market power in logs because a dead weight welfare

[8]The Fréchet derivative is a matrix whose rows correspond to the derivatives of the matching elements of $\mathbf{G}$ with respect to the matching variables, at a particular point in space. For example the (2,2) element of $D_{(\mathbf{a},\mathbf{b})}\mathbf{G}$ is the derivative of the second element of $\mathbf{G}$ with respect to $\hat{p}^e_x$, at some point $(\mathbf{a},\mathbf{b}) = (\mathbf{a}_0,\mathbf{b}_0)$.

loss is created. Efficiency will occur if free trade prevails; however, either economy can improve its individual welfare through the use of trade restrictions (all else constant). I take the view in this section that each country acts in their own self interest without the possibility of co-operation. This may not be a good assumption if retaliatory restrictions on other goods will be implemented by a country's trading partner if they exercise their market power. Forest products are among the few goods not included in NAFTA, meaning the United States would have few options for retaliation.[9]

The notion of a benevolent social planner is used here to compute the home economy's first best optimal. The notation '$\diamond\diamond$' is used to denote the first best allocation in the two country model.[10]

Definition The home economy first best allocation, $(m^{\diamond\diamond}, y_d^{\diamond\diamond}, x^{\diamond\diamond})$, is a general equilibrium allocation such that $U(m^{\diamond\diamond}, y_d^{\diamond\diamond}) \geq U(m, y_d)$.

Notice that the first best optimal is restricted to be a general equilibrium. The social planner's problem is to maximize $U(m, y_d)$ subject to

$$m + \hat{p}_y y_d \leq f(x^e)\hat{p}_y + \tilde{x}^e \hat{p}_x^e(\tilde{x}^e) - c(y^e), \tag{17}$$

where $x^e = \overline{x} - \tilde{x}^e$ and $y^e = f(x^e)$. The necessary conditions for a first best optimal are:

$$\begin{aligned} \frac{U_{p_y}}{U_m} &= \hat{p}_y & (18) \\ \hat{p}_x^e - \tilde{x}^e f'(x)^2 c''(y) &= f'(x)\left(\hat{p}_y - c'(y^e)\right) & (19) \end{aligned}$$

Equations (12) and (13) must coincide with equations (18) and (19), respectively, for the general equilibrium to be the home economy's first best. If $\alpha = 0$ then equations

[9] Commodity lumber is not an option as trade is already restricted.

[10] Notation $\diamond\diamond$ is used because a single $\diamond$ was used in the small economy model.

(12) and (18) are identical, and if $\tau = \tilde{x}^e f'(x)^2 c''(y)/\hat{p}_x^e$ equations (13) and (19) will coincide.

Proposition 4. *If $\tilde{x}^e > 0$ and $\hat{p}_x^e > 0$, and the home economy first best allocation is attained, then $\alpha = 0$, $\tilde{x}^e = \frac{1}{3}\left(\overline{x} - \hat{\overline{x}}\right)$ and $\tau = \tilde{x}^e f'(x)^2 c''(y)/\hat{p}_x^e$. If $\alpha > 0$ then the home first best allocation cannot be attained.*

Proof. When $\alpha = 0$ equations (12), (14) and (18) are satisfied always, but if $\alpha \neq 0$ equations (12) and (18) are never simultaneously satisfied. Equations (13) and (19) are simultaneously satisfied only if $\tau = \tilde{x}^e f'(x)^2 c''(y)/\hat{p}_x^e$ whenever $\tilde{x}^e > 0$. At that τ, equation (15) is satisfied only if $\tilde{x}^e = \frac{1}{3}\left(\overline{x} - \hat{\overline{x}}\right)$. Thus, the necessary conditions for general equilibrium and the home economy optimal allocation are met when $\tilde{x}^e > 0$ only at the hypothesized α, τ and $\tilde{x}^e$. When $\tilde{x}^e = \frac{1}{3}\left(\overline{x} - \hat{\overline{x}}\right)$, the general equilibrium condition $x + \hat{x} = \overline{x} + \hat{\overline{x}}$ is satisfied trivially. □

Though Proposition 1 provides a nice solution for how the first best can be attained when there is no tax on lumber exports, it also tells us that the first best cannot be attained when such a tax is levied. Like in the small open economy case, a second best allocation will be the basis for an optimal log export policy in the presence of a lumber export tax. A modified version of Ruffin's Lemma provides a useful necessary condition for the second best allocation.

Definition Given $\alpha \neq 0$, the home economy second best allocation, $(m^{ss}, y_d^{ss}, x^{ss})$, is a general equilibrium allocation such that $U(m^{ss}, y_d^{ss}) \geq U(m, y_d)$.

Lemma 2. *If $U_{y_d}/U_m = k\hat{p}_y$, where $k \neq 1$ is a constant, then a necessary condition for a second best allocation is*

$$\hat{p}_x^e - \tilde{x}^e f'(x)^2 c''(y) = f'(x)\left(\hat{p}_y - c'(y^e)\right). \tag{19}$$

Proof. The social planner's problem is to maximize $U(m, y_d)$ subject to equation (17) and $U_{y_d}/U_m = k\hat{p}_y$, or equivalently:

$$\max_{\{m,y_d,x\}} L^p = U(m, y_d) + \lambda_1 \left[f(x^e)\hat{p}_y + \tilde{x}^e\hat{p}^e_x(\tilde{x}^e) - c(y^e) - m - \hat{p}_y y_d\right] + \lambda_2 \left[k\hat{p}_y - \frac{U_{y_d}}{U_m}\right].$$

The first-order necessary conditions for a maximum are equality in the constraints and:

$$\nabla L^p = \begin{pmatrix} U_m - \lambda_1 - \lambda_2 \left(U_{y_d m}U_m - U_{mm}U_{y_d}\right)/U_m^2 \\ U_{y_d} - \hat{p}_y\lambda_1 - \lambda_2 \left(U_{y_d y_d}U_m - U_{m y_d}U_{y_d}\right)/U_m^2 \\ \lambda_1 \left(f'(x)\hat{p}_y - \hat{p}^e_x - \tilde{x}^e(\partial\hat{p}^e_x/\partial\tilde{x}^e) - f'(x)c'(y)\right) \end{pmatrix} = \mathbf{0}.$$

The third condition implies equation (19), since $\lambda_1 \neq 0$ unless there is waste. □

3.3 Optimal Home Log Export Policy

A complete definition of an optimal home economy log export policy is given.

Definition An exogenously chosen optimal home log export policy, τ^{**}, is such that given $\alpha > 0$, $(m^*, y^*_d, x^*) = (m^{ss}, y^{ss}_d, x^{ss})$, and given $\alpha = 0$, $(m^*, y^*_d, x^*) = (m^{\diamond\diamond}, y^{\diamond\diamond}_d, x^{\diamond\diamond})$ so that $U(m^*(\tau^{**}), y^*_d(\tau^{**})) \geq U(m^*(\tau), y^*_d(\tau))\ \forall\tau$.

Optimal home economy log export policy requires that the first best allocation be attained if it is feasible, and the second best allocation be attained if it is not.

Theorem 2. *For the two country model and a given arbitrary lumber export tax α, if $\tilde{x}^e > 0$ and $\hat{p}^e_x > 0$ then the optimal home economy log export tax is*

$$\tau^{**} = \frac{f'(x)\hat{p}_y}{\hat{p}^e_x}\alpha + \frac{1}{3}\left(\overline{x} - \hat{\overline{x}}\right)\frac{f'(x)^2 c''(y)}{\hat{p}^e_x}. \tag{20}$$

Proof. When $\alpha > 0$, optimal home policy is the second best allocation. To attain the second best, equations (12) through (14) must hold by definition, and so must equation

(19) by Lemma 2. When $\tilde{x}^e > 0$ and $\hat{p}_x^e > 0$ equations (13) and (19) are satisfied simultaneously if and only if

$$\tau = \frac{f'(x)\hat{p}_y}{\hat{p}_x^e}\alpha + \frac{\tilde{x}^e f'(x)^2 c''(y)}{\hat{p}_x^e}.$$

For equation (15) to be satisfied as well it must be the case that $\tilde{x}^e = \frac{1}{3}\left(\overline{x} - \hat{\overline{x}}\right)$ giving the result when $\alpha > 0$. When $\alpha = 0$, optimal home policy is the first best allocation, which is achieved when equation (21) holds by Proposition 4. □

Corollary 2. *Suppose $\tilde{x}^e > 0$ and $\hat{p}_x^e > 0$, then $\tilde{x}^e = \frac{1}{3}\left(\overline{x} - \hat{\overline{x}}\right)$ when $\tau = \tau^{**}$.*

Proof. Included in the proofs of Theorem 2 and Proposition 4. □

Theorem 2 is an intriguing result as the components of the home economy optimal log export tax are the optimal tax for a small open economy (Theorem 1) added to the tax that exists if the home economy attains the first best (Proposition 4). The first term on the RHS of equation (20) is equivalent to τ^*, the optimal log export policy for a small economy, which guaranteed that logs were allocated identically to free trade. The second term of equation (20) is necessary for the home economy to achieve the first best allocation when feasible. When the first best is achieved the home economy has maximized its welfare by manipulating its terms of trade. Putting these ideas together gives a nice interpretation of τ^{**}: the first term shifts logs away from exports to the allocation that would prevail under free trade, and the second term shifts additional logs away from exports to the allocation that maximizes the home country's welfare by taking advantage of market power.

This notion is reinforced by Corollary 2, which says that under the optimal log export tax the equilibrium quantity of logs exported is invariant to the lumber export tax. One third of the difference between home and foreign harvests is exported, in contrast to one half the difference being exported under free trade.

The intuition for the result is much like the intuition for the optimal log export tax of a small economy. The social planner is forced to choose the best policy they can, given the conditions of general equilibrium. To do this they want to enact a policy that will make the home representative agent's budget set as large as possible, but they are unable to change the slope of its boundary. This amounts to maximizing the intercept of equation (17), rearranged so that m is a function of y_d. Using the derivatives in equation (16), this is easily shown to coincide with Theorem 2.

3.4 The Optimal and Dumont and Wright's Policy - Again

As noted previously, the primary motivation for this paper is to better understand a particular policy recommendation. In the small economy case comparison of Dumont and Wright's (2006) recommendation to the optimal policy was cumbersome, although a meaningful comparison was made. In the case where the exporting economy has international market power in logs, no definitive comparison can be made. The best that can be accomplished is to note that over at least some range of the variables Dumont and Wright's policy implies a larger log export tax than the optimal policy, but there is no certainty regarding the entire range.

Proposition 5. *Suppose* $f'(x) < (1-\alpha)$*, then* $\tau^{**} < \overline{\tau}$ *if*

$$\frac{1}{3}\left(\overline{x} - \hat{\overline{x}}\right) < \frac{(1-\alpha-f'(x))\hat{p}_y\alpha}{f'(x)^2 c''(y)}. \tag{21}$$

Proof. Following from Proposition 2, $\overline{\tau} > (1-\alpha)(\hat{p}_y/\hat{p}_x)\alpha$, thus $\tau^{**} < \overline{\tau}$ if

$$(1-\alpha-f'(x))\hat{p}_y\alpha > \tilde{x}^e f'(x)^2 c''(y).$$

By Corollary 2, $\tilde{x}^e = (1/3)\left(\overline{x} - \hat{\overline{x}}\right)$, thus the above inequality can be re-arranged to give the result. The condition $f'(x) < (1-\alpha)$ ensures that the RHS of (21) is a positive quantity. $\square$

The right hand side of inequality (21) is a positive constant, given the conditions of Proposition 5, and there is an open interval of possible values for $\tilde{x}^e$ that will satisfy the inequality. Whether or not the numbers in that interval are large enough to be realistic is an empirical matter.

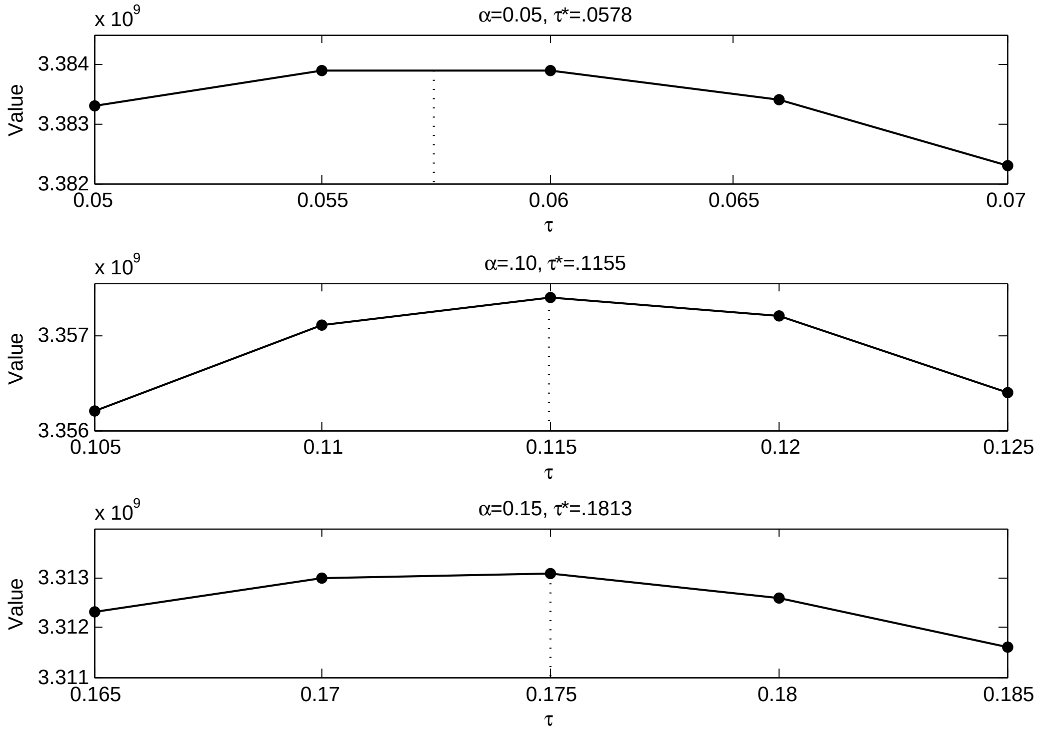
The possibility that $\overline{\tau}$ is optimal for some parameter values cannot be refuted, but it can be said that it is not optimal always. That is, when the home economy has market power in logs, there are circumstances under which Dumont and Wright's recommendation cannot be optimal.

4 An Empirical Example

A spatial price equilibrium (SPE) forest product trade model is calibrated to show that the optimal log export taxes derived above hold in this commonly used partial equilibrium framework. The model of Abbott, Stennes and van Kooten (2008) provides a simple and convenient method to capture the derived demand relationship between logs and lumber within the SPE framework. This model may be a good approximation to the model developed above because producers' surplus can be re-interpreted as consumption of the second good, m, whose price is unaffected by changes in the forest industry. To simplify matters, the multiple world regions in the model are replaced by a perfectly competitive international market into which British Columbia exports forest products, and the time dimension of the model is dropped. International demand is perfectly elastic and international prices are set equal to average world prices. The British Columbia parameters are calibrated as in Abbott, Stennes and van Kooten. Appendix 2 describes this model in greater detail.

The optimal log export tax is determined heuristically for three different lumber export taxes. This is accomplished by fixing α and varying τ by 0.005%, then plotting the total forest value (the sum of consumers' and producers' surpluses and export tax revenue) against τ. Figure 2 provides these plots for $\alpha = \{0.05, 0.10, 0.15\}$, for which the optimal log export taxes implied by equation (9) are $\tau^* = \{0.0578, 0.1155, 0.1733\}$. The export taxes implied by equation (9) are correct for the SPE model within the 0.005% allowed for in the experiment, suggesting that the general equilibrium results derived above are generalizable to this commonly used partial equilibrium model.

Figure 2: SPE Optimal Log Export Taxes



5 Conclusion and Discussion

Several key findings were established in this study. When the B.C. forest industry faces a tax on lumber exports the welfare maximizing log export policy is a positive export tax. The arguments for free trade are irrelevant in these instances because it is unattainable. How large the log export tax should be depends on the extent to which B.C. has power in international log markets.

The log export tax recommended by Dumont and Wright (2006) is generally not optimal. If B.C. is a price-taker in international log markets, their tax would be excessive and create inefficient distortions. The optimal policy is larger than the lumber export tax, as Dumont and Wright argue, but by a smaller magnitude than they suggest. If B.C. does have market power in international log markets, the Dumont and Wright policy over taxes log exports when the quantity of log exports is small enough. For larger log export totals this may not be the case. As noted in the introduction, log export policy in B.C. is not shaped by efficiency alone. It is possible that Dumont and Wright's (2006) policy is based on more than efficiency considerations; however, because their recommended tax is zero when no lumber export tax is levied, there is reason to be skeptical.

Regardless of Dumont and Wright's motivations, distributional considerations will continue to play a role in B.C. log export policy. Extensions of the present study should consider re-distribution related characteristics, such as monopolistic wage contracts and government spending. Another extension could investigate in a game theoretic model the potential role for log export policy as a bargaining tool in the Canada-U.S. softwood lumber trade dispute.

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6 Appendix 1 - Sufficient Conditions

The purpose of this appendix is to show that the many necessary conditions described in the paper are describing maxima. In general the assumed quasiconcavity of the utility function guarantees a maximum, however, as one of the choice variables does not appear directly in the utility function this may not be clear. For completeness the problems are described in detail. The representative agent's utility maximization problem - the small economy model agent, and both two country model agents - can be generically written as follows.

$$\begin{array}{ll} \max_{m,y_d,x} & U(m,y_d) \\ S.T. & m + p_y y_d \leq f(x)p_y + (\overline{x} - x)p_x - c(f(x)) + T \end{array}$$

Here the prices will depend on which agent is being considered, but they are generally taken as exogenous parameters by the representative agent, and thus are immaterial. Foreign "hat" variables could be substituted with no change to the results. A Lagrangian formed to solve the problem:

$$\max_{\{m,y_d,x,\lambda\}} L = U(m,y_d) + \lambda \left[f(x)p_y + (\overline{x} - x)p_x - c(f(x)) + T - m - p_y y_d \right].$$

The necessary conditions for an interior maximum (or any critical value) is $\nabla L = \mathbf{0}$, which gives rise to the necessary conditions given in the text. The sufficient condition for any critical point to be a maximum is that the determinants of the border preserving leading principle minors of the Hessian matrix of order $k = 2, 3$ have signs $(-1)^k$. The

Hessian matrix of the given Lagrangian is:

$$\mathbf{H}L = \begin{pmatrix} U_{mm} & U_{my} & 0 & -1 \\ U_{ym} & U_{yy} & 0 & -p_y \\ 0 & 0 & -f'(x)c''(y) & 0 \\ -1 & -p_y & 0 & 0 \end{pmatrix}.$$

The determinant of the principle minor of order $k = 2$, formed by deleting row 3 and column 3, is

$$|\mathbf{H}L_2| = \begin{vmatrix} U_{mm} & U_{my} & -1 \\ U_{ym} & U_{yy} & -p_y \\ -1 & -p_y & 0 \end{vmatrix} = 2U_{ym}p_y - U_{yy} - U_{mm}p_y^2 > 0.$$

$|\mathbf{H}L_2|$ must be positive because $U(m, y_d)$ is quasiconcave. The determinant of the principle minor of order $k = 3$ is the determinant of $\mathbf{H}L$ itself, which is $-f'(x)^2c''(y)\,|\mathbf{H}L_2| < 0$. Thus the determinants meet the criteria that they have signs $(-1)^k$ and the critical points analyzed above are indeed describing maxima.

The social planner's problem for efficiency in the small open economy will have the same bordered Hessian as the representative agent's problem (allowing for a different p_y), however, in the two economy model it will not. In the two country model L_x differed from the agent's problem in that market power in logs was accounted for. What this means for the Hessian matrix is that the (3,3) element will be $-2f'(x)^2c''(y)$, but all else will be the same as the representative agent's problem. Obviously this will not change the signs of the principle minors, and maxima are attained for those problems as well.

The sufficient conditions for the problems used to prove Lemmas 1 and 2 are now derived. The Lagrangian and first order conditions for the problem of second best in the small economy case are given in the proof of Lemma 1. The bordered Hessian of that Lagrangian is 5×5. To make it fit nicely on the page, two changes of variable are made.

$$A = \tfrac{U_{y_d m} U_m - U_{mm} U_{y_d}}{U_m^2} \qquad B = \tfrac{U_{y_d y_d} U_m - U_{m y_d} U_{y_d}}{U_m^2}$$

The bordered Hessian is:

$$\mathbf{H}L = \begin{pmatrix} U_{mm} - \lambda_2 \frac{\partial A}{\partial m} & U_{m y_d} - \lambda_2 \frac{\partial A}{\partial y_d} & 0 & -1 & A \\ U_{y_d m} - \lambda_2 \frac{\partial B}{\partial m} & U_{y_d y_d} - \lambda_2 \frac{\partial B}{\partial y_d} & 0 & -\hat{p}_y & B \\ 0 & 0 & -f'(x)^2 c''(y) & 0 & 0 \\ -1 & -\hat{p}_y & 0 & 0 & 0 \\ A & B & 0 & 0 & 0 \end{pmatrix}.$$

For a maximum the leading border preserving principle minors of order 2 and 3 must have positive and negative determinants, respectively. Deleting the third row and column and taking the determinant results in:

$$|\mathbf{H}L_2| = \hat{p}_y^2 A^2 + B^2 - 2AB\hat{p}_y.$$

This can be factored into the more convenient form $|\mathbf{H}L_2| = (\hat{p}_y A - B)^2 > 0$. The determinant of the principle minor of order 3 is $f'(x)c''(y)(\hat{p}_y A - B)^2 < 0$, and thus any solution implied by the necessary conditions is a maximum. In the two country model the solution to the second best problem is also a maximum since $|\mathbf{H}L_2| = (\hat{p}_y A - B)^2 > 0$ and $|\mathbf{H}L_3| = 2f'(x)c''(y)(\hat{p}_y A - B)^2 < 0$, very similar to the small economy case.

7 Appendix 2 - SPE Model

Spatial price equilibrium models solve for a competitive partial equilibrium in a particular market by determining the allocation that maximizes total surplus. A series of calibrated functions describing demand and costs were employed in the model used in §4:

$$\begin{array}{lrcl} \textit{B.C. lumber demand} & p_y & = & a_i - a_s y_d \\ \textit{lumber marginal cost} & c'(y) & = & b_s y \\ \textit{log marginal cost} & z'(x) & = & c_x \\ \textit{international lumber demand} & \hat{p}_y(\tilde{y}) & = & \hat{p}_y \\ \textit{international log demand} & \hat{p}_x(\tilde{x}) & = & \hat{p}_x \end{array}$$

The parameters used to calibrate these equations, given in Table A1, are borrowed from Abbott, Stennes and van Kooten (2008). Total surplus is calculated by summing consumer benefits and subtracting resource and taxation costs as follows:

$$\begin{aligned} S(y_d, \tilde{y}, \tilde{x}) &= \int_0^{y_d} (a_i - a_s y) dy + (1-\tau)\hat{p}_x \tilde{x} + (1-\alpha)\hat{p}_y \tilde{y} \\ &\quad - \int_0^{y_d+\tilde{y}} b_s y dy - ((1/\phi)(y_d + \tilde{y}) + \tilde{x}) c_x, \end{aligned}$$

where ϕ is the lumber recovery factor, i.e. $f'(x)$. This is equivalent to:

$$\begin{aligned} S(y_d, \tilde{y}, \tilde{x}) &= (a_i - \phi c_x) y_d + ((1-\alpha)\hat{p}_y - \phi c_x)\tilde{y} + ((1-\tau)\hat{p}_x - c_x)\tilde{x} \\ &\quad - \tfrac{1}{2}((a_s + b_s) y_d^2 + b_s \tilde{y}^2) - b_s y_d \tilde{y}. \end{aligned}$$

Total surplus, as measured by $S(y_d, \tilde{y}, \tilde{x})$, is maximized subject to a single constraint:

$$(1/\phi) y_d + (1/\phi)\tilde{y} + \tilde{x} \leq \overline{x}.$$

This constraint says that the sum of logs manufactured into lumber or sold internationally cannot exceed the annual allowable cut. This problem was solved using the QUADPROG function in MATLAB.

Table A1: Calibration Parameters

Parameter	Value
a_i	235
a_s	$5.7 \times 10^{-}6$
b_s	$1.3636 \times 10^{-}6$
c_x	51
$\overline{x}$	8.4×10^7
ϕ	0.5
$\hat{p}_y$	201
$\hat{p}_x$	87